

# solutions

```
K.<d> = QQ[ ]
E = EllipticCurve([1+d-d^2, d^2-d^3, d^2-d^3,
0, 0])
E
```

Elliptic Curve defined by  $y^2 + (-d^2+d+1)*x + (-d^3+d^2)*x^2$  over Univariate Polynomial Field

```
E.short_weierstrass_model()
```

Elliptic Curve defined by  $y^2 = x^3 + (-27*d^8+324*d^7-1134*d^6+1512*d^5-945*d^4+3(54*d^{12}-972*d^{11}+6318*d^{10}-19116*d^9+30780*11988*d^5+9396*d^4-2484*d^3-810*d^2+324*d+54$  Polynomial Ring in  $d$  over Rational Field

```
show(E.short_weierstrass_model())
```

$$y^2 = x^3 + (-27d^8 + 324d^7 - 1134d^6 + 1512d^5 - 945d^4 -$$

```
for d in [1..1000]:
    if is_fundamental_discriminant(d):
        K = QuadraticField(d)
        if K.class_number() == 5:
            print "d = ", d
            break
```

$d = 401$

```

for d in [-1,-2,...,-1000]:
    if is_fundamental_discriminant(d):
        K = QuadraticField(d)
        if K.class_number() == 5:
            print "d = ", d
            break

```

d = -47

```

R.<x> = QQ[ ]
for d in [1..500]:
    f = x^3 + d
    if not f.is_irreducible(): continue
    K = NumberField(f, 'a')
    if K.class_number() == 5:
        print K
        break

```

Number Field in a with defining polynomial x

```

def E(a):
    return EllipticCurve([0,(a-1),1,-a,0])

for a in [0..80]:
    print a, E(a).rank()

```

```

0 0
1 1
2 2
3 2
4 3
5 2
6 2
7 3

```

8 3  
9 3  
10 2  
11 3  
12 3  
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```
74 4
75 3
76 3
77 3
78 3
79 5
80 3
```

```
# so smallest are: 0,1,2,4,16,79.
```