Graph Theory Project

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Oct. 22-25, 2017

1 Project 1

There is significant interest in studying methods for link prediction and network evolution in networks (both directed and undirected, and both weighted and unweighted). For example, a simple strategy is to count the number of common neighbors between nodes in a simple graph, and select the pairs of non-adjacent nodes with the highest 'score' to be the best link for prediction. Or we could consider how many paths of length 3 (or 4 or 5 ...) between nodes, and think of a way to weight shorter paths more heavily. Alternatively, we let the network represent a resistor network, and consider the effective resistance between nodes as a score function for determining the 'best' new links. We will focus on implementing these and other scoring methods for graphs and digraphs. For example, in the case of undirected graphs, any pseudoinverse can be used to compute effective resistance. For directed graphs, though, this is not the case. Thus, we can implement the standard effective resistance for (weighted and unweighted) graphs, and other network distances using various pseudoinverses for (weighted and unweighted) digraphs.

We will also work on creating methods to generate important graphs and digraphs, like directed and undirected triangular grids and directed and undirected 2-trees (linear, branched, bent, pinwheels), and use these as our working examples for the link prediction algorithms discussed above.

2 Project 2

One interesting way to find *some* of the eigenvalues of a (di)graph with a nontrivial automorphism group, is by creating a partition of the vertices and using these to generate a smaller digraph. It turns out that this smaller digraph's eigenvalues are always a subset (as a multiset) of those of the original, larger (di)graph! These so-called equitable partitions seem like magic (and are already implemented in SAGE) but beg the question: What about the rest of the eigenvalues? My coauthors and I published a series of papers showing how to create a series of these small digraphs whose eigenvalues give the full spectrum of the original (di)graph. Using equitable decompositions (this extension of equitable partitions) to find eigenvalues is a simple algorithm that hasn't yet been implemented in SAGE.